

Name: _____ Per: _____

AP Physics C

Semester 1 - Mechanics

Unit 4

Work, Power &

Conservation of

Energy Workbook

Unit 4 - Work, Power, & Conservation of Energy

Supplements to Text Readings from
Fundamentals of Physics by Halliday, Resnick, & Walker **Chapter 7 & 8**

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Unit 4 - Objectives & Assignments

TEXT: *Fundamentals of Physics* by Halliday, Resnick, & Walker, Chapter 7 & 8

I. Work and the Work-Energy Theorem (Ch. 7)

a. Students should understand the definition of work so they can:

- (1) Calculate the work done by a specified constant force on a body that undergoes a specified displacement.
- (2) Relate the work done by a force to the area under a graph of forces as a function of position, and calculate this work in the case where the force is a linear function of position.
- (3) Use integration to calculate the work performed by a force $F(x)$ on a body that undergoes a specified displacement in one dimension.
- (4) Use the scalar product operation to calculate the work performed by a specified constant force $F(x)$ on a body that undergoes a displacement in a plane.

b. Students should understand the work-energy theorem so they can:

- (1) State the theorem precisely, and prove it for the case of motion in one dimension.
- (2) Calculate the change in kinetic energy or speed that results from performing a specified amount of work on a body.
- (3) Calculate the work performed by the net force, or by each of the specified change in speed or kinetic energy.
- (4) Apply the theorem to determine the change in a body's kinetic energy and speed that results from the application of specified forces, or to determine the force that is required in order to bring a body to rest in a specified distance.

II. Power (Ch. 7)

a. Students should understand the definition of power so they can:

- (1) Calculate the power required to maintain the motion of a body with constant acceleration (e.g., to move a body along a level surface, to raise a body at a constant rate, or to overcome friction for a body that is moving at a constant speed).
- (2) Calculate the work performed by a force that supplies constant power, or the average power supplied by a force that performs a specified amount of work.
- (3) Prove that the relation $P = F \cdot v$ follows from the definition of work, and apply this relation in analyzing particle motion.

III. Conservative Forces and Potential Energy (Ch. 8)

a. Students should understand the concept of a conservative force so they can:

- (1) State two alternative definitions of "conservative force", and explain why these definitions are equivalent.
- (2) Describe two examples each of conservative forces and nonconservative forces.

- b. Students should understand the concept of potential energy so they can:
- (1) State the general relation between force and potential energy, and explain why potential energy can be associated only with conservative forces.
 - (2) Calculate a potential energy function associated with a specified one-dimensional force $F(x)$.
 - (3) Given the potential energy function $U(x)$ for a one-dimensional force, calculate the magnitude and direction of the force.
 - (4) Write an expression for the force exerted by an ideal spring and for the potential energy stored in a stretched or compressed spring.
 - (5) Calculate the potential energy of a single body in a uniform gravitational field.
 - (6) Calculate the potential energy of a system of bodies in a uniform gravitational field.
 - (7) State the generalized work-energy theorem and use it to relate the work done by nonconservative forces on a body to the changes in kinetic and potential energy of the body.

IV. Conservation of Energy (Ch. 8)

- a. Students should understand the concepts of mechanical energy and of total energy so they can:
- (1) State, prove, and apply the relation between the work performed on a body by nonconservative forces and the change in a body's mechanical energy.
 - (2) Describe and identify situations in which mechanical energy is converted to other forms of energy.
 - (3) Analyze situations in which a body's mechanical energy is changed by friction or by a specified externally applied force.
- b. Students should understand conservation of energy so they can:
- (1) Identify situations in which mechanical energy is or is not conserved.
 - (2) Apply conservation of energy in analyzing the motion of bodies that are moving in a gravitational field and are subject to constraints imposed by strings or surfaces.
 - (3) Apply conservation of energy in analyzing the motion of bodies that move under the influence of springs.
 - (4) Apply conservation of energy in analyzing the motion of bodies that move under the influence of other specified one-dimensional forces.
- c. Students should be able to recognize and solve problems that call for application both of conservation of energy and Newton's Laws.

Unit 4 Homework

Chapter 7 #6, 8, 14, 18, 23, 25, 31, 33, 36, 37, 43, 45, 49

Chapter 8 #5, 8, 9, 10, 33, 36, 39, 40, 53, 55, 59, 62

Work, Power & Energy Problems

Math Review - Vector Dot Products

$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta = A_x B_x + A_y B_y + A_z B_z$ (multiplying the parallel components of two vectors)
where θ is the angle between vectors \mathbf{A} and \mathbf{B} .

Given the following vectors

$$\mathbf{A} = 7\mathbf{i} - 8\mathbf{j}$$

$$\mathbf{B} = -3\mathbf{i} + 2\mathbf{j}$$

$$\mathbf{C} = 12, 133^\circ$$

$$\mathbf{D} = 6, 270^\circ$$

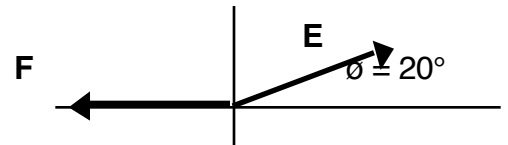
1. Find $\mathbf{A} \cdot \mathbf{B}$ ¹

2. Find $\mathbf{A} \cdot \mathbf{D}$ ² and the angle³ between \mathbf{A} and \mathbf{D}

3. Find $\mathbf{C} \cdot \mathbf{D}$ ⁴

4. Find $\mathbf{B} \cdot \mathbf{C}$ ⁵ and the angle⁶ between \mathbf{B} and \mathbf{C}

5. Given the vectors \mathbf{E} and \mathbf{F} in the diagram on the right, where $|\mathbf{E}| = 7$ and $|\mathbf{F}| = 4$, find $\mathbf{F} \cdot \mathbf{E}$ ⁷.



¹ -37

² 48

³ 41.2°

⁴ -52.7

⁵ 42.1

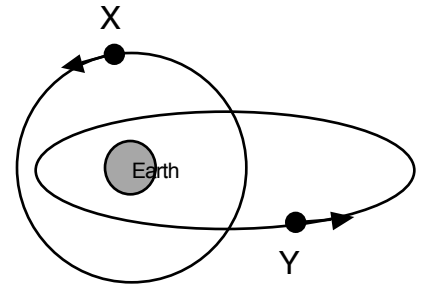
⁶ 13.3°

⁷ -26.3

1. Earth & Satellites

The sketch at the right shows two satellites orbiting about the Earth. Satellite X has a circular orbit and satellite Y has an elliptical orbit.

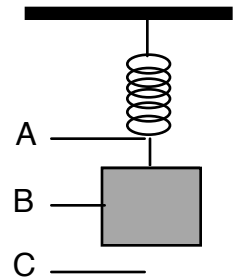
- During which portion(s) of its orbit is the gravitational force doing positive, negative, or zero work on satellite X?
- During which portion(s) of its orbit is the gravitational force doing positive, negative, or zero work on satellite Y?
- Explain you answers to (a) and (b) in terms of the relative directions of the gravitational force and the displacement vectors.
- What is happening to the kinetic energy of the satellites when the work done is positive? ...negative? ...zero? In each case, is the kinetic energy increasing, decreasing, or constant?



2. Mass on a Spring

A mass, m , oscillates up and down on the end of a spring as shown on the right. There are two forces on m , the force of gravity, F_g , and the force due to the spring, F_s .

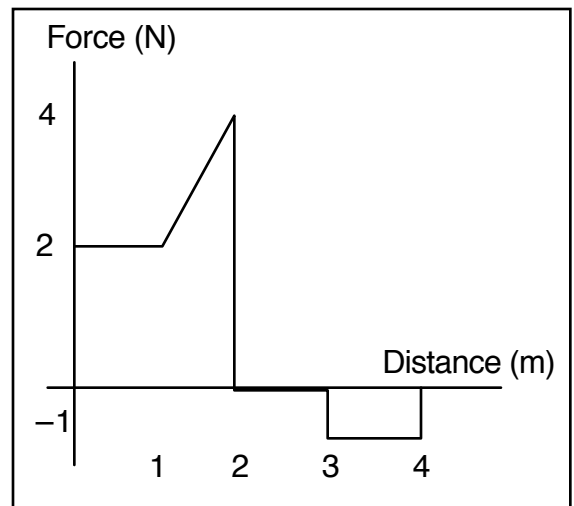
- During what part(s) of the oscillation is F_g doing positive work on m ? ...negative work?
- During what part(s) of the oscillation is F_s doing positive work on m ? ...negative work?
- During what part(s) of the oscillation is the NET force doing positive work on m ? ...negative work? ... zero work?



3. Interpreting Force vs. Distance Graphs

The graph at the right represents the net force acting on a $2/3$ kg object plotted as a function of position. At $t = 0$ the object is at the origin and is moving with velocity 2 m/s in the $+x$ direction.

- Find the net work done on the object after it has moved 1 meter.
- Find the change in object's kinetic energy as it moves from $x = 0$ to $x = 3$ meters.
- Find the net work done on the object as it moves from $x = 3$ m to $x = 4$ m.
- Find the speed of the object after it has moved 4 meters from its initial position.



Unit 4 - Work, Power, Conservation of Energy

4. Space Probes, Work, & Kinematics

Suppose a mechanical probe of mass 5×10^4 kg and initial velocity of 1.1×10^4 m/s is out in space. Its engine is turned on and it exerts a force of 4×10^5 N on the probe that causes a displacement of 2.5×10^6 m. What is its final velocity⁸ at the end of the displacement?

5. Pulling Crates

To pull a 50 kg crate across a frictionless ice floor, a worker applies a force of 210 N, directed 20° above the horizontal. As the crate moves 3 m, what work is done on the crate by

- the worker's force⁹
- the weight of the crate¹⁰
- the normal force exerted by the floor on the crate¹¹
- What is the total work done¹² on the crate?

⁸ 1.27×10^4 m/s

⁹ 592 J

¹⁰ 0

¹¹ 0

¹² 592 J

6. Using vector products

Given $\mathbf{F} = (7\mathbf{i} - 6\mathbf{j})$ Newtons

- Find the work done¹³ by \mathbf{F} on an object as it causes the object to move from the origin to a position given by $\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j})$ meters.
- If \mathbf{F} is the only force acting on the object, does the kinetic energy of the object increase, decrease, or remain the same as it moves from the origin to \mathbf{r} ? If so, by how much¹⁴?
- Find the average power¹⁵ if it took 0.6 seconds for the object to move from the origin to \mathbf{r} .

¹³ -45 J

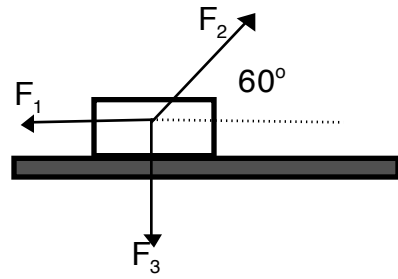
¹⁴ decrease by 45 J

¹⁵ -75 W

7. Work by Multiple Forces

The figure below shows 3 forces applied to a greased trunk that moves leftward by 3 m over a frictionless floor where $F_1 = 5 \text{ N}$, $F_2 = 9 \text{ N}$, $F_3 = 3 \text{ N}$.

- During the displacement, what is the net work¹⁶ done on the trunk by the three forces?
- During the displacement, does the KE of the trunk increase or decrease?



¹⁶ 1.5 J

8. Work and Velocity

A force acts on a 3 kg particle-like object in such a way that the position of the object as a function of time is given by $x = 3t - 4t^2 + t^3$, with x in meters and t in seconds. Find the work done¹⁷ on the object by the force from $t = 0$ to $t = 4$ s. (Hint: What are the speeds at those times?)

¹⁷ 528 J

9. Work by Angled Forces

A constant force of magnitude 10 N makes an angle of 150° (measured counterclockwise) with the direction of increasing x as the force acts on a 2 kg object. How much work¹⁸ is done on the object by the force as the object moves from the origin to the point with position vector

$$\mathbf{r} = (2 \text{ m})\mathbf{i} - (4 \text{ m})\mathbf{j}?$$

¹⁸ -37 J

10. Work on an Inclined Plane

A 45 kg block of ice slides down a frictionless incline 1.5 m long and 0.91 m high. A worker pushes up against the ice, parallel to the incline, so that the block slides down at a constant speed.

- Find the force¹⁹ exerted by the worker.
- How much work²⁰ is done on the block by the worker's force?
- How much work²¹ is done on the block by the weight of the crate?
- How much work²² is done on the block by the normal force?
- What is the total work done²³ on the block?

¹⁹ 265 N

²⁰ -400 J

²¹ 400 J

²² 0

²³ 0

11. Work by Lifting

A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is $g/10$.

- How much work²⁴ is done on the astronaut by the force of the helicopter?
- How much work²⁵ is done on the astronaut by her weight?
- What is the kinetic energy²⁶ of the astronaut just before she reaches the helicopter?
- What is the speed²⁷ of the astronaut just before she reaches the helicopter?

²⁴ $1.2 \times 10^4 \text{ J}$

²⁵ $-1.1 \times 10^4 \text{ J}$

²⁶ $1.1 \times 10^3 \text{ J}$

²⁷ 5.3 m/s

12. Work by Gravity on a Projectile

A projectile of mass m is thrown with initial speed v_0 at an angle θ above the horizontal.

- What is the kinetic energy²⁸ of the projectile at the highest point of its trajectory?
- What is the work done²⁹ on the projectile by the gravitational force between its initial position and the apex of its trajectory?

²⁸ $m(v_0 \cos \theta)^2 / 2$

²⁹ $mv_0^2(\cos^2 \theta - 1) / 2$

13. Work by Mystery Force

The force on a particle is $F(x) = F_0 (x/x_0 - 1)$. Find the work³⁰ done by the force in moving the particle from $x = 0$ to $x = 2x_0$.

³⁰ 0

14. Work by Various Forces

Find the work done by the following forces acting on an object along the x direction.

- a)³¹ $F(x) = 3x^5 - 5$ -from $x = 4$ to $x = 7$ m
- b) $F(x) = \sin x$ -from³² $x = 0^\circ$ to $x = 90^\circ$
-from³³ $x = 90^\circ$ to $x = 180^\circ$
-from³⁴ $x = 0^\circ$ to $x = 180^\circ$
- c)³⁵ $F(x) = 3x^2 + x^3$ -from $x = -5$ to $x = 5$
- d)³⁶ $F(y) = 1/(y^2 + 36)^{1/2}$ -from $y = 4$ to $y = 8$

³¹ 56761.5 J

³² 57.3 J

³³ 57.3 J

³⁴ 114.6 J

³⁵ 250 J

³⁶ 0.47 J

15. Work by an Exponentially Increasing Force

A force \mathbf{F} in the direction of increasing x acts on an object moving along the x axis. If the magnitude of the force is $F(x) = 10e^{-x/2}$ N where x is in meters. Find the work³⁷ done by \mathbf{F} as the object moves from $x = 0$ to $x = 2$ m.

³⁷ 12.6 J

16. Work by Quadratic Force

The magnitude of the net force acting on a 2 kg object moving along the x axis is given by

$$F(x) = 3x^2 - 4x + 5 \quad \text{where } x \text{ is in meters and } F \text{ in Newtons.}$$

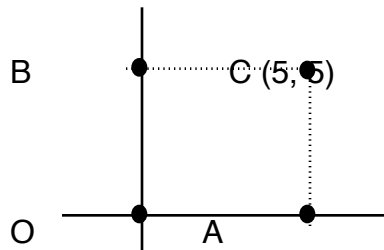
If the speed of the object is 5 m/s at $x=1$ m, what is its speed³⁸ at $x=3$ m?

³⁸ $3\sqrt{5}$ m/s

17. Work Along Different Paths

A force of $F = (2yi + x^2j)$ N acts on a particle along various paths.
Determine the work done by the force F

- a) along path OAC³⁹
- b) along path OBC⁴⁰
- c) along path OC⁴¹



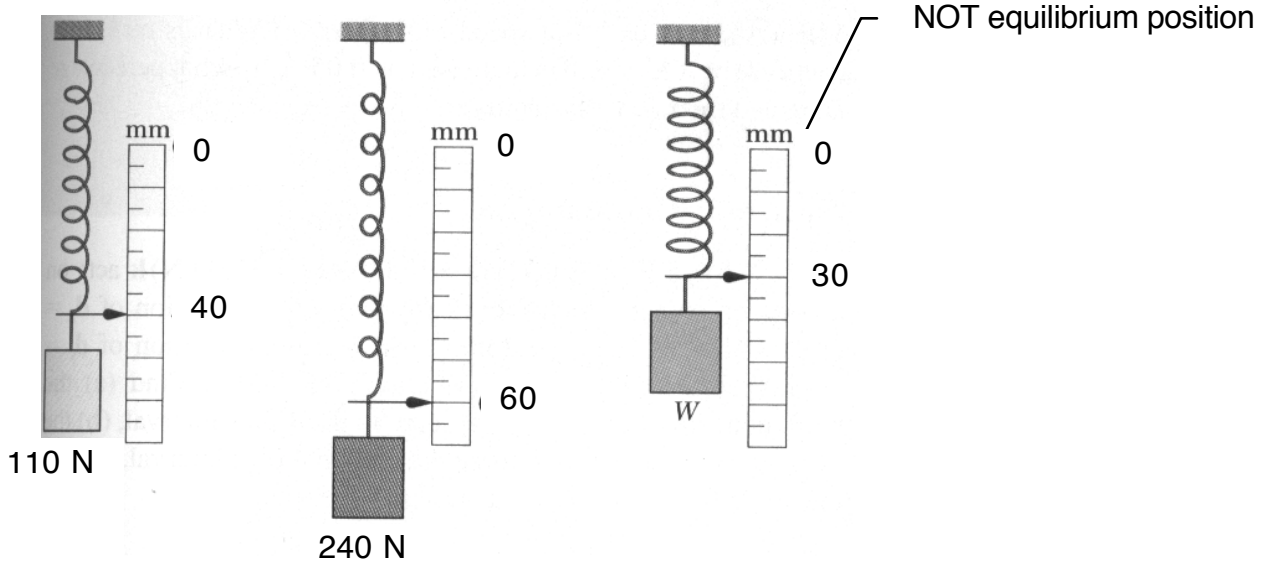
³⁹ 125 J

⁴⁰ 50 J

⁴¹ 175 J

18. Hanging on Springs

A spring with a pointer attached is hanging next to a scale marked in millimeters. Three different weights are hung from the spring, in turn, as shown below. (a) If all weight is removed from the spring, which mark⁴² on the scale will the pointer indicate? (b) What is the weight⁴³ W ?



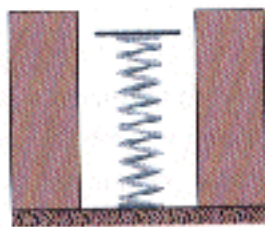
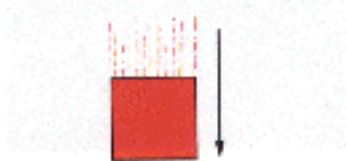
⁴² 23 mm

⁴³ 45 N

19. Work with Gravity and Springs

A 250 g block is dropped onto a vertical spring with spring constant $k = 2.5 \text{ N/cm}$ (below). The block becomes attached to the spring, and the spring compresses 12 cm before momentarily stopping.

- While the spring is being compressed, what work is done on the block
 (a) by its weight⁴⁴ and (b) by the spring force⁴⁵? (c) What is the speed⁴⁶ of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression⁴⁷ of the spring?



⁴⁴ $2.9 \times 10^{-1} \text{ J}$

⁴⁵ -1.8 J

⁴⁶ 3.5 m/s

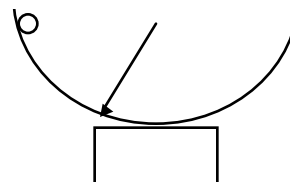
⁴⁷ 21 cm

Conservation of Energy Problems

1. Potential Energy

As shown below, a 2 g ice flake is released from the edge of a bowl whose radius is 22 cm. The flake-bowl contact is frictionless.

- How much work⁴⁸ is done on the flake by its weight during the flake's descent to the bottom of the bowl?
- What is the change in the potential energy⁴⁹ of the flake-Earth system during the descent?
- If that potential energy is taken to be zero at the bottom of the bowl, what is its value⁵⁰ when the flake is released?
- If, instead, the potential energy is taken to be zero at the release point, what is its value⁵¹ when the flake reaches the bottom of the bowl?



⁴⁸ 4.31×10^{-3} J

⁴⁹ -4.31×10^{-3} J

⁵⁰ 4.31×10^{-3} J

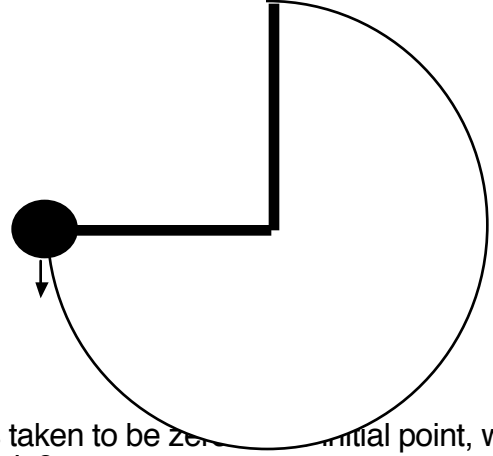
⁵¹ -4.31×10^{-3} J

Unit 4 - Work, Power, Conservation of Energy

2. Energy Transformation in Swinging Ball

The figure on the right shows a ball with mass m attached to the end of a thin rod with length L and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle. The rod is held in the horizontal position as shown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertically upward position, with zero speed there. How much work is done on the ball by its weight from the initial point to

- the lowest point?
 - the highest point?
 - the point on the right that is directly level with the initial point?
- d) If the gravitational potential energy of the ball-Earth system is taken to be zero at the initial point, what is its value when the ball reaches those three points, respectively?



3. Elastic Potential Energy

A spring with a spring constant of 3200 N/m is initially stretched until the elastic potential energy is 1.44 J ($U = 0$ for no stretch). What is the change in the elastic potential energy if the initial stretch is changed to a) ⁵² a stretch of 2 cm, b) ⁵³ a compression of 2 cm, and c) ⁵⁴ a compression of 4 cm?

⁵² -0.8 J

⁵³ -0.8 J

⁵⁴ 1.1 J

4. Conservative Forces

A force \mathbf{F} that is directed along an x axis acts on a particle as the particle moves from $x = 1$ m to $x = 4$ m and then back to $x = 1$ m. What is the net work done on the particle by \mathbf{F} for the round trip if the values of the force during the outward and the return trips are

- a)⁵⁵ 3 N and -3 N,
- b)⁵⁶ 5 N and 5 N,
- c)⁵⁷ $2x$ and $-2x$, Here, x is in meters and f is in Newtons
- d)⁵⁸ $3x^2$ and $3x^2$
- e)⁵⁹ In which situation(s) is the force conservative?

⁵⁵ 18 J

⁵⁶ 0 J

⁵⁷ 30 J

⁵⁸ 0 J

⁵⁹ b) & d)

5. Spring Guns

A 5 g marble is fired vertically upward using a spring gun. The spring must be compressed 8 cm if the marble is to reach a target 20 m above the marble's position on the compressed spring.

- What is the change in the gravitational potential energy⁶⁰ of the marble-Earth system during the 20 m ascent?
- What is the change in the elastic potential energy⁶¹ of the spring during its launch of the marble?
- What is the spring constant⁶² of the spring?

⁶⁰ 0.98 J

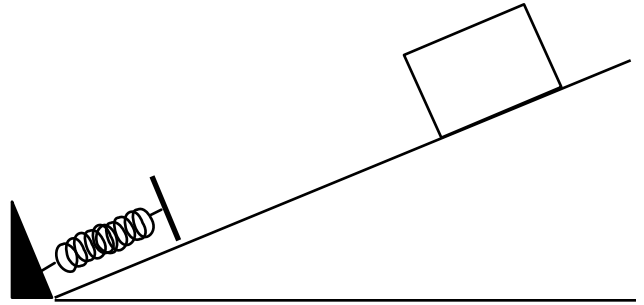
⁶¹ -0.98 J

⁶² 310 N/m

6. Pinball Launcher Type Thing

As shown on the right, a 12 kg block is released from rest on an inclined plane at $\theta = 30^\circ$. Below the block is a spring that can be compressed 2 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm.

- How far⁶³ has the block moved down the incline to this stopping point?
- What is the speed⁶⁴ of the block just as it touches the spring?



⁶³ 0.35 m

⁶⁴ 1.7 m/s

Unit 4 - Work, Power, Conservation of Energy

7. Kinetic Energy of a Projectile

A 50 g ball is thrown from a window with an initial velocity of 8 m/s at an angle 30° above the horizontal. Using energy methods, determine

- the KE⁶⁵ of the ball at the apex of its parabolic path.
- its speed⁶⁶ when it is 3 m below the window.
- Does the answer to b) depend on either the mass of the ball or the initial angle?

⁶⁵ 1.2 J

⁶⁶ 11 m/s

8. Compressing Springs

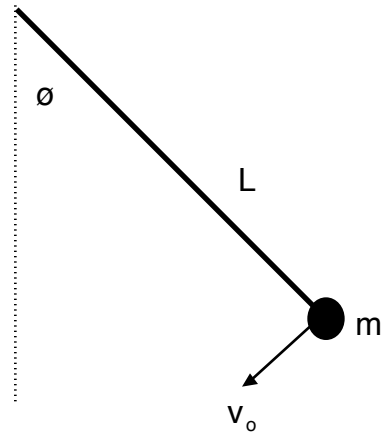
A 2 kg block is dropped from a height of 40 cm onto a spring of spring constant $k = 1960 \text{ N/m}$. Find the maximum distance⁶⁷ the spring is compressed.

⁶⁷ 0.1 m

9. Energy in a Pendulum

A pendulum of length L has a mass on the end. The bob has speed v_0 when the cord makes an angle θ_0 with the vertical.

- Derive the expression for the speed⁶⁸ of the bob when it is in its lowest position.
- What is the least value⁶⁹ that v_0 can have if the pendulum is to swing down then up to a horizontal position?
- What is the least value⁷⁰ that v_0 can have if the pendulum is to swing down then up to vertical position with the cord remaining straight?



⁶⁸ $(v_0^2 + 2gL(1 - \cos\theta_0))^{1/2}$

⁶⁹ $(2gL\cos\theta_0)^{1/2}$

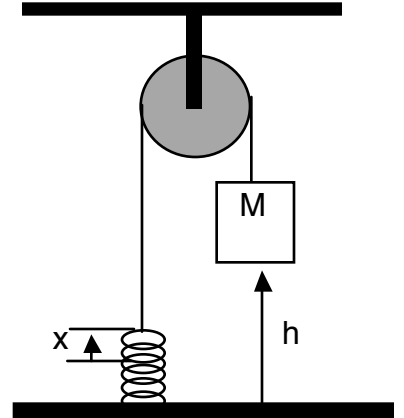
⁷⁰ $(gL(3 + 2\cos\theta_0))^{1/2}$

10. Pulleys and Springs

In the set-up shown to the right is a cord strung over a light pulley of negligible friction. A mass M is tied to one end of a cord and the other end is attached to a light spring fixed to the floor.

Given: $M = 0.4 \text{ kg}$, $k = 20 \text{ N/m}$, $h = 1 \text{ m}$ when $x = 0$

- What are the values⁷¹ of x and h when the system is in equilibrium?
- If M is lifted to $h = 1 \text{ m}$ and dropped, what is its kinetic energy⁷² as it passes through its equilibrium position?
- When $x = 0.3 \text{ m}$, what is the speed⁷³ of M ?
- What is the maximum value⁷⁴ of x ?



⁷¹ $x=0.2\text{m}$, $h=0.8\text{m}$

⁷² 0.4 J

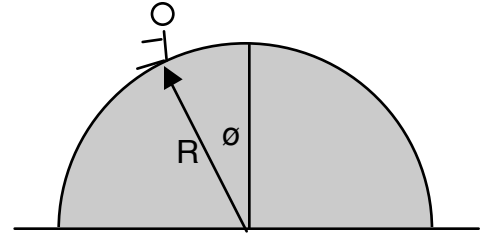
⁷³ 1.22 m/s

⁷⁴ 0.4 m

11. Kid on a Slide

A kid having mass m is seated atop a humongous hemispherical ball of ice. He is given a tiny push whereupon he begins to slide down the ice.

- a) In terms of m , g , R , and θ , determine each of the following while the kid is in contact with the ice.
- The kid's kinetic energy⁷⁵
 - The kid's centripetal acceleration⁷⁶
 - The tangential acceleration⁷⁷ of the kid
- b) Show that the kid loses contact with the ice when he is a vertical distance of $2R/3$ above the ground.



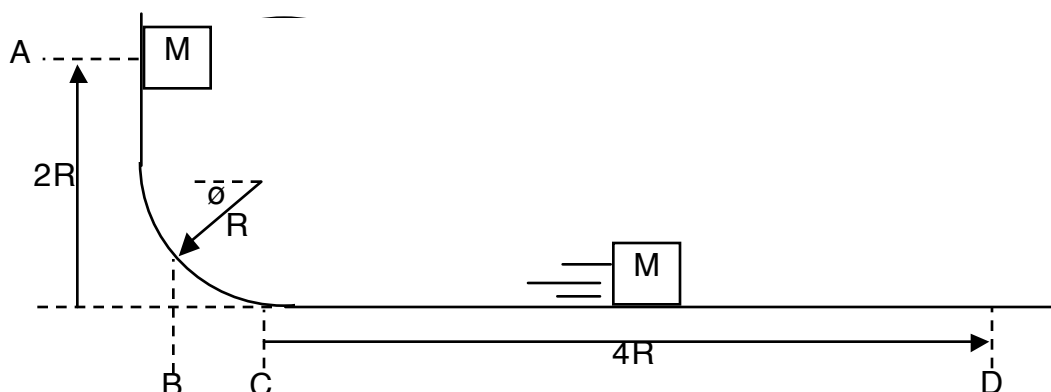
⁷⁵ $mgR(1-\cos\theta)$

⁷⁶ $2g(1-\cos\theta)$

⁷⁷ $g\sin\theta$

12. Drop Zone

The situation below is similar to an amusement ride called the Drop Zone. A box full of people with total mass M is released from some high position A where it falls basically frictionless to point C. Once the box reaches horizontal motion, constant frictional forces bring it to a stop at point D.



Answer the following in terms of M , R , and other appropriate constants.

a) Given that $\theta = 30^\circ$ when the block reaches point B, find the speed⁷⁸ of the block at B.

b) Find the speed⁷⁹ of the block as it reaches point C.

c) Determine the work done⁸⁰ on the block by friction between points C and D.

⁷⁸ $\sqrt{3gR}$

⁷⁹ $2\sqrt{gR}$

⁸⁰ $-2MgR$

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- d) Find the magnitude of the friction force⁸¹ on the block.
- e) Find the coefficient⁸² of friction between the block and the track.
- f) If the time required for the block to move between points C and D is T , determine the rate⁸³ at which energy is being dissipated as heat.
- g) Determine the magnitude⁸⁴ of the normal force on the block as it passes through point B. (Hint: Find the centripetal force and use it with the result in part a.)

⁸¹ $Mg/2$

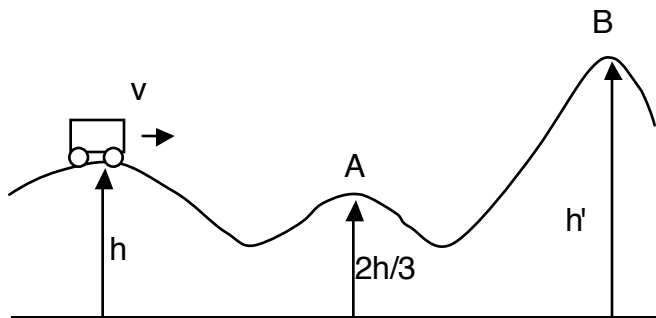
⁸² 0.5

⁸³ $-2MgR/T$

⁸⁴ $7Mg/2$

13. Roller Coaster

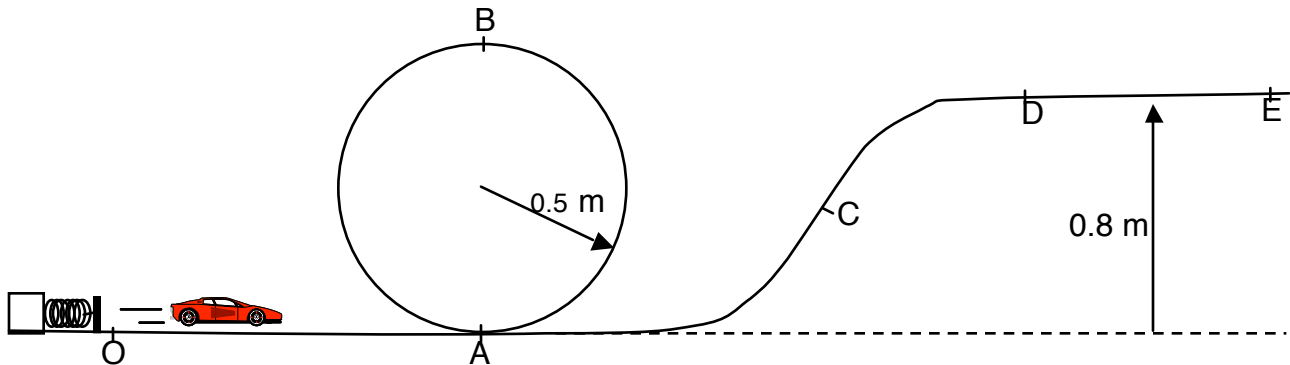
A frictionless roller coaster is given an initial speed v_0 at a height h as shown below. The radius of curvature of the track at point A is R .



- Find the maximum value of v_0 necessary in order that the roller coaster NOT leave the track at point A.
- Using the value of v_0 calculated in part a), determine the value of h' necessary if the roller coaster is to make it just to point B.

Review of Conservation of Energy

The sketch above shows a track consisting of a level section, OA, a loop-the-loop, and a hill, ACD, rising to a level section, DE. A 0.2 kg toy car is free to move along the track. At the left end of the track, a spring having a spring constant of 500 N/m is mounted horizontally on a rigid, fixed wall. Consider the track-car-spring-earth system to be conservative.



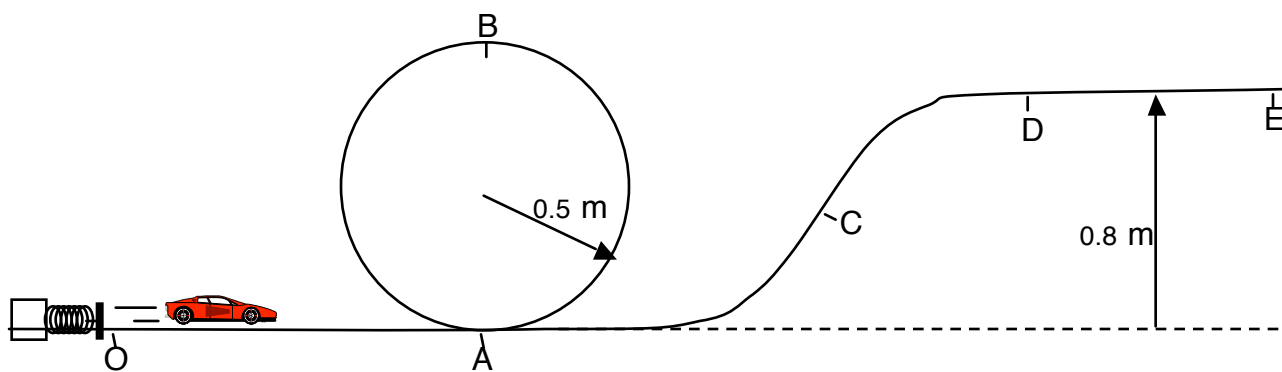
- a) If the car just barely comes out of contact with the track at point B, what is its speed⁸⁵ at this point?
Hint: find the centripetal force on the car at point B.
- b) The car is given a shove to the left at point D of sufficient magnitude that it arrives at point B with the speed calculated in part a. What is the car's kinetic energy⁸⁶ just after the shove?
- c) If level OA is chosen as the reference level for gravitational potential energy and the speed of the car at point B is that found in part a, what is the mechanical energy⁸⁷ of the system?

⁸⁵ 5 m/s

⁸⁶ 0.9 J

⁸⁷ 2.5 J

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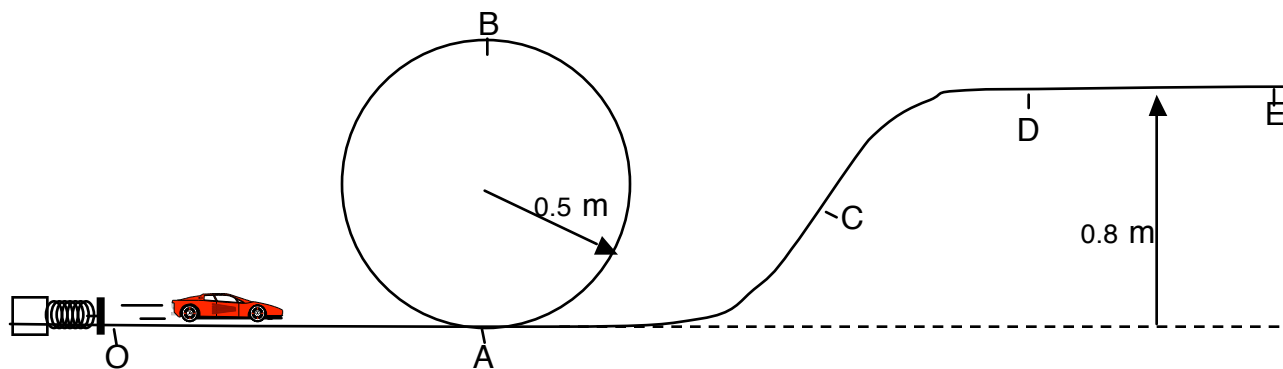
d) What is the speed⁸⁸ of the car when it passes through point A?

e) The car travels to the left from point A comes in contact with and compresses the spring. What is the maximum compression⁸⁹ of the spring?

⁸⁸ 5 m/s

⁸⁹ 0.1 m

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f) The car is then pushed to the right by the spring, passing again through the loop and up the slope to point D. After point D, the track becomes a bed of sand bringing the car to rest at point E. How much work⁹⁰ is done on the car as it move from D to E?

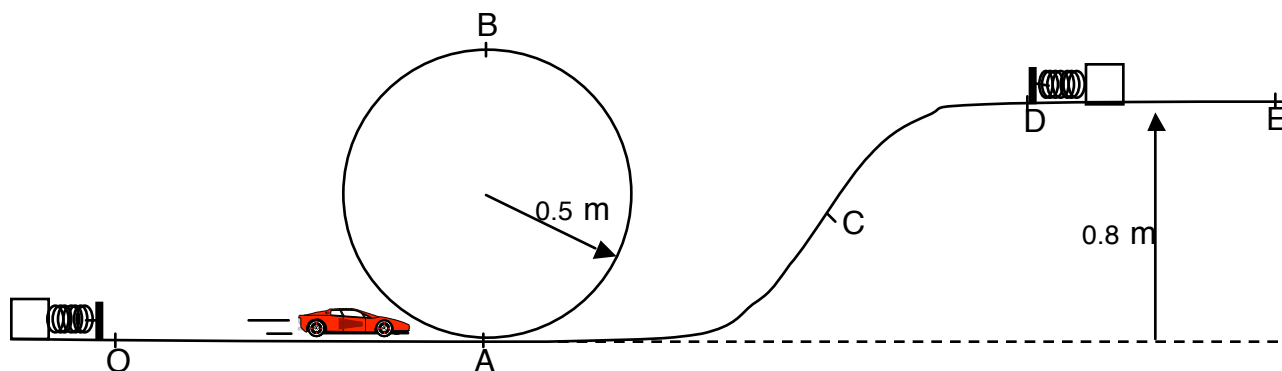
g) If the sand exerts a constant force of 2 N on the car, what is the distance⁹¹ between point D and E?

⁹⁰ -0.9 J

⁹¹ 0.45 m

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Instead of the sand after D, let there be a spring mounted as shown in the sketch below. This spring has a constant of 100 N/m. Let all other conditions remain the same.



- h) When the car arrives at point D, it begins to compress the spring. At the instant the spring is compressed 0.1 m, what is the kinetic energy⁹² of the car?

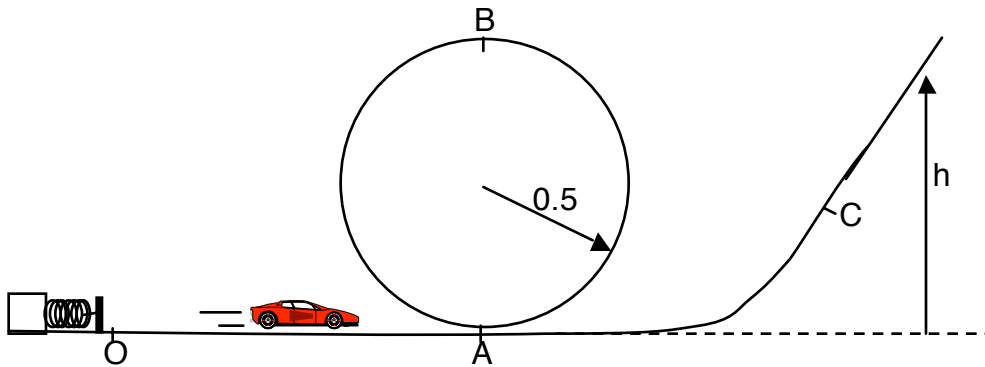
- i) What is the maximum compression⁹³ of this spring?

⁹² 0.4 J

⁹³ 0.13 m

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Suppose that instead of the level section and spring the track continued to slope upward part point C as shown below. Let all other conditions remain the same.



j) What is the maximum height⁹⁴ above the reference level the car reaches?

k) Suppose someone reaches in as the car is moving from A toward C and gives the car a shove in the direction of its motion thereby going 0.5 J of work on the car. What is the maximum height⁹⁵ the car will reach this time? If the shove is in a direction opposite to its direction of motion, what maximum height⁹⁶ will the car reach?

⁹⁴ 1.25 m

⁹⁵ 1.5 J

⁹⁶ 1 m

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- l) Under the conditions stated in the first part of part k, suppose that the car rolled back down into the loop. When it reaches point B again, what is its speed⁹⁷?

- m) Under the conditions stated in the first part of part k, after the car rolled out of the loop and moved against the spring, compressing it, and momentarily coming to rest, what will be the maximum compression⁹⁸ of the spring this time?

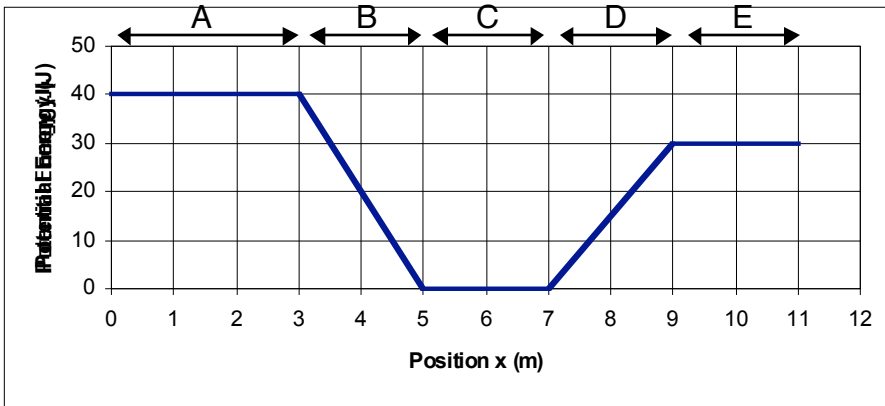
⁹⁷ 10 m/s

⁹⁸ 0.11 m

More About Potential Energy

1. Potential Energy Graphs

1. The graph below shows the potential energy $U(x)$ of a 2 kg object plotted as a function of its position x .



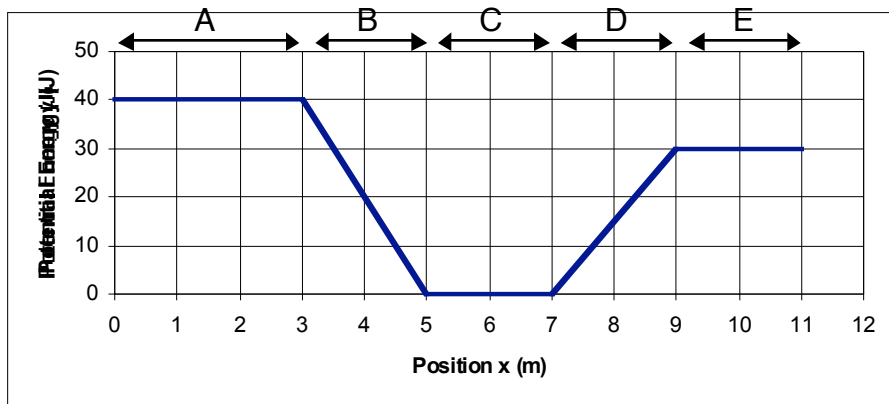
- a) What is the magnitude and direction⁹⁹ of the conservative force acting on the object at $x = 4$, 6, and 8 m?
- b) In which of the labeled region(s)¹⁰⁰ is the force on the object in the same direction as its velocity?
...in the opposite direction as its velocity?
- c) In which region(s)¹⁰¹ is the object in equilibrium?

⁹⁹ 20 N, +x direction; 0; 15 N, -x direction

¹⁰⁰ B, D

¹⁰¹ A, C, E

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d) What is the acceleration¹⁰² of the object at $x = 2$, 3.5 , and 7.5 m?

e) In which region(s)¹⁰³ is the work being done on the object positive? _____
 negative? _____ zero? _____

f) Suppose the kinetic energy of the object is 20 J when it is at $x = 6$ m, what is the total mechanical energy¹⁰⁴ of the system?

¹⁰² 0, +10 m/s², -7.5 m/s²

¹⁰³ positive: B, negative: D, zero: A, C, E

¹⁰⁴ 20 J

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- g) Suppose the kinetic energy of the object is 20 J when it is at $x = 6$ m, what is the object's kinetic energy¹⁰⁵ at $x = 4$? At $x = 8$ m?
- h) What is the speed¹⁰⁶ of the object at $x = 4$ m? ... at $x = 8$ m?
- i) What are the maximum and minimum values¹⁰⁷ of x to which the object can move?

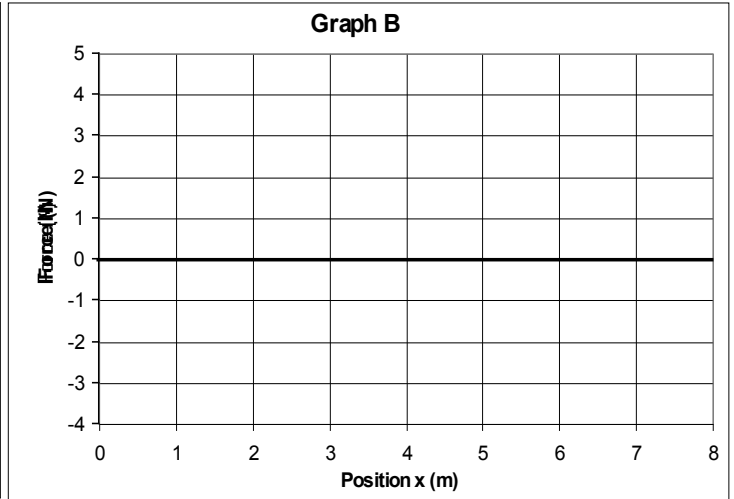
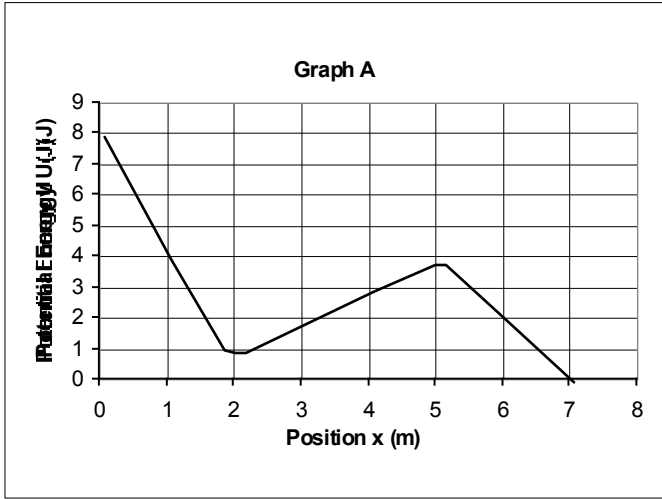
¹⁰⁵ 0, 5 J

¹⁰⁶ 0, $\sqrt{5}$ m/s

¹⁰⁷ $4 \text{ m} < x < 8.4 \text{ m}$

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2. Graph A below shows the potential energy $U(x)$ of an object plotted as a function of its position x .



Suppose the object has total mechanical energy of 4 J.

b) Determine the kinetic energy¹⁰⁹ of the object at $x = 2$ m. $x = 4$ m?

c) Can the object reach the position $x = 0.5$ m? Explain.

d) Can the object reach the position $x = 5$ m? Explain.

e) On the graph B above, plot the graph of the force $F(x)$ acting on the object as a function of its position x for $0 < x < 7$ m.

¹⁰⁸ 2 m, stable; 5 m unstable

¹⁰⁹ 3 J, 1 J

2. Potential Energy Functions

The potential energy function for a system is given by $U = ax^2 - bx$, where a and b are constants.

a) Find the force F_x associated with this potential energy function?

b) At what value of x is the force zero?

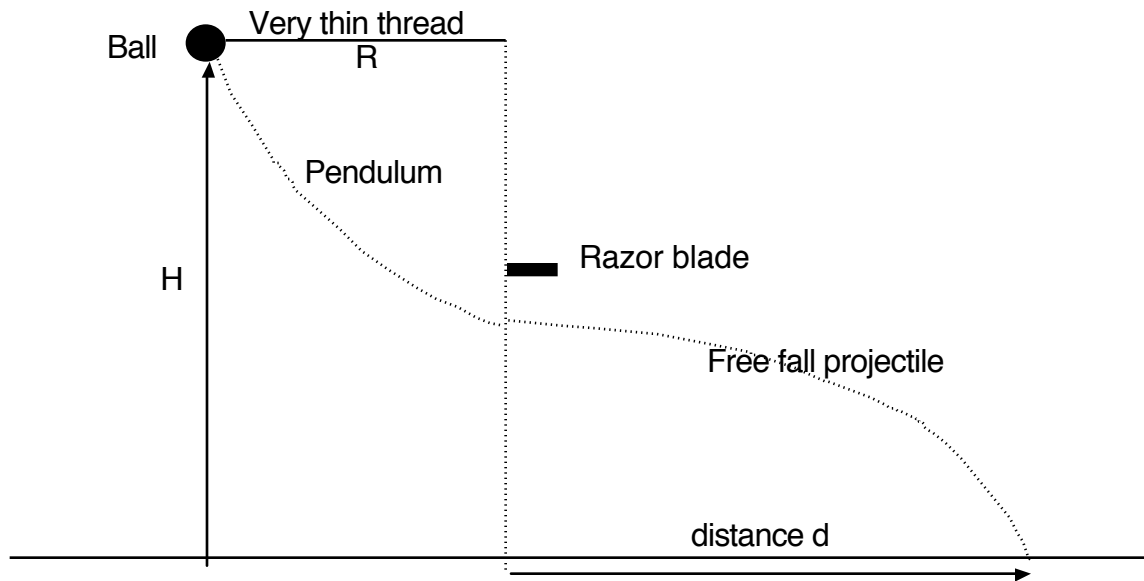
Pendulum & Projectile - Conservation of Energy Lab

Purpose: To use the Principle of Conservation of Energy & Kinematics to predict and verify through experimentation where the ball shown below will land on the table for any given starting height.

Pre-lab theory: (To be completed as homework BEFORE doing the experiment)
Use the Principle of Conservation of Energy & Kinematics determine the expression for the distance d as a function of vertical height H for the system shown below.

Use your expression for $d(H)$ to make an appropriate data table for the experiment.

Have your teacher check your expression and data table before proceeding with the lab.



Procedures & Data:

Once your expression for $d(H)$ and data table has been checked, proceed with doing the experiment.
Your group decides the procedures and adequate number of trial data for good analysis.

Analysis & Conclusions:

Show me that you have analyzed your results worthy of AP Physics students including sample calculations, % errors, sources of systematic error, etc.